DIOPHANTINE ANALYSIS.

86. Proposed by B. F. FINKEL, A. M., M.Sc., Professor of Mathematics and Physics, Drury College, Springfield, Mo.

Prove that the congruence $x^2=1457\equiv 0(\mathrm{mod}2389)$ is not possible.

A solution of this problem is given in Mathews' *Theory of Numbers*, page 42. ED.

87. Proposed by LON C. WALKER, A.M., Professor of Mathematics, Petaluma High School, Petaluma, Cal.

Find three numbers in arithmetical progression the sum of whose cubes is a cube.

Solution by the PROPOSER.

Let tx, ty, tz represent the three required numbers. Then we have

$$t^3x^3+t^3y^3+t^3z^3=w^3\text{; or, } x^3+y^3+z^3=w^3/t^3=v^3\text{......(1), assume.}$$

Transposing (1), we have to satisfy the equation

$$v^3-x^3-y^3=z^3\text{......(2).}$$

Put $x=a(r-q)$, $y=aq$, $z=a[p-(rq^2/p^2)]$, and $v=ap$. Then we have, after dividing by a^3,

$$p^3-(r-q)^3-q^3=(p-\frac{rq^2}{p^2})^3\text{......(3).}$$

Whence, by involution and reduction,

$$r=\frac{3p^3q}{p^3+q^3}.$$

Now take $a=p^3+q^3$, and we have

$$x=q(2p^3-q^3),\ y=q(p^3+q^3),\ z=p(p^3-2q^3),\ v=p(p^3+q^3),$$

where p and q may have any values subject to the condition $p^3>2q^3$.

If we take $p=2$, $q=1$, after dividing all by 3, we have, $3^3+4^3+5^3=6^3$, the least three positive integral numbers that satisfy conditions of the problem.

[See *Mathematical Magazine*, page 154, from whence the above solution was copied.]

88. Proposed by L. C. WALKER, A. M., Professor of Mathematics, Petaluma High School, Petaluma, Cal.

Find three square numbers in harmonical progression.

Solution by HARRY S. VANDIVER, Bala, Pa.

Solutions of this problem have been given on pages 82-83 of Vol. VII, of the MONTHLY, substantially as follows: